

Step Response of RC Circuits

1. Objectives

- Measure the output waveform of simple RC circuits excited by step functions.
- Calculate and measure various timing parameters of switching waveforms (time constant, delay time, rise time, and fall time) common in computer systems.
- Compare theoretical calculations and experimental data, and explain any discrepancies.

2. Reference

The step response of RC circuits is covered in the textbook. Review the appropriate sections, look at signal waveforms, and review the definition and formula for the time constant.

Review the laboratory exercise one for the usage of laboratory instruments.

3. Circuits

Figure 1 shows the first-order RC circuit whose step response will be studied in this lab.

Figure 2 shows two sections of the first-order RC circuit connected in series to illustrate a simple technique to model computer bus systems (PCI bus, SCSI bus, etc.).

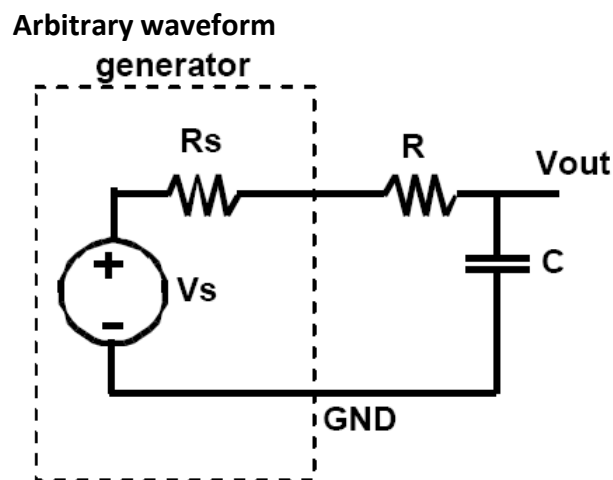


Figure 1. Simple RC circuit.

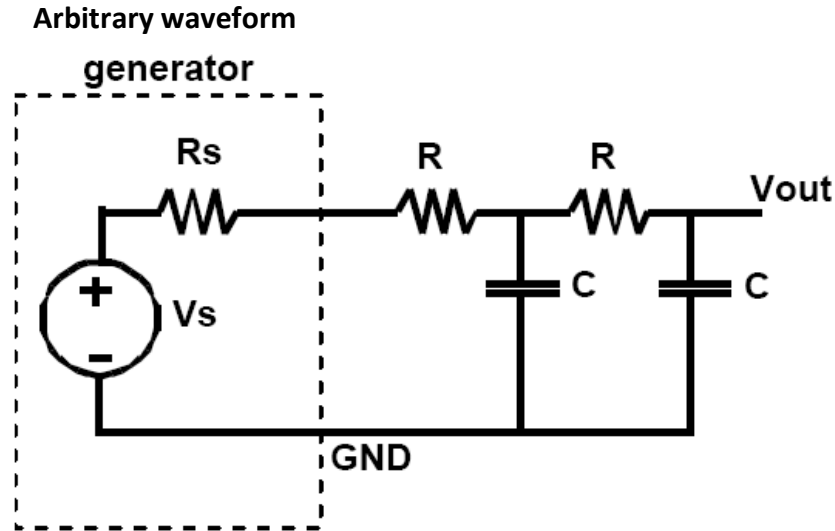


Figure 2. Two cascaded RC sections.

4. Components and specifications

| Quantity | Description | Comments |
|----------|------------------------|----------|
| 1 | 50 Ω resistor | |
| 2 | 10 K Ω resistor | |
| 1 | 27 K Ω resistor | |
| 2 | 0.01 μ F capacitor | |

5. Discussion

5.1 Step response and timing parameters

The step response of a simple RC circuit, illustrated in Figure 3, is an exponential signal with time constant $\tau = RC$. Besides this timing parameter, four other timing parameters are important in describing how fast or how slow an RC circuit responds to a step input. These timing parameters are marked in Figure 3, at three voltage levels:

- The 10%-point is the point at which the output voltage is 10% of the maximum output voltage.
- The 50%-point is the point at which the output voltage is 50% of the maximum output voltage.
- The 90%-point is the point at which the output voltage is 90% of the maximum output voltage.

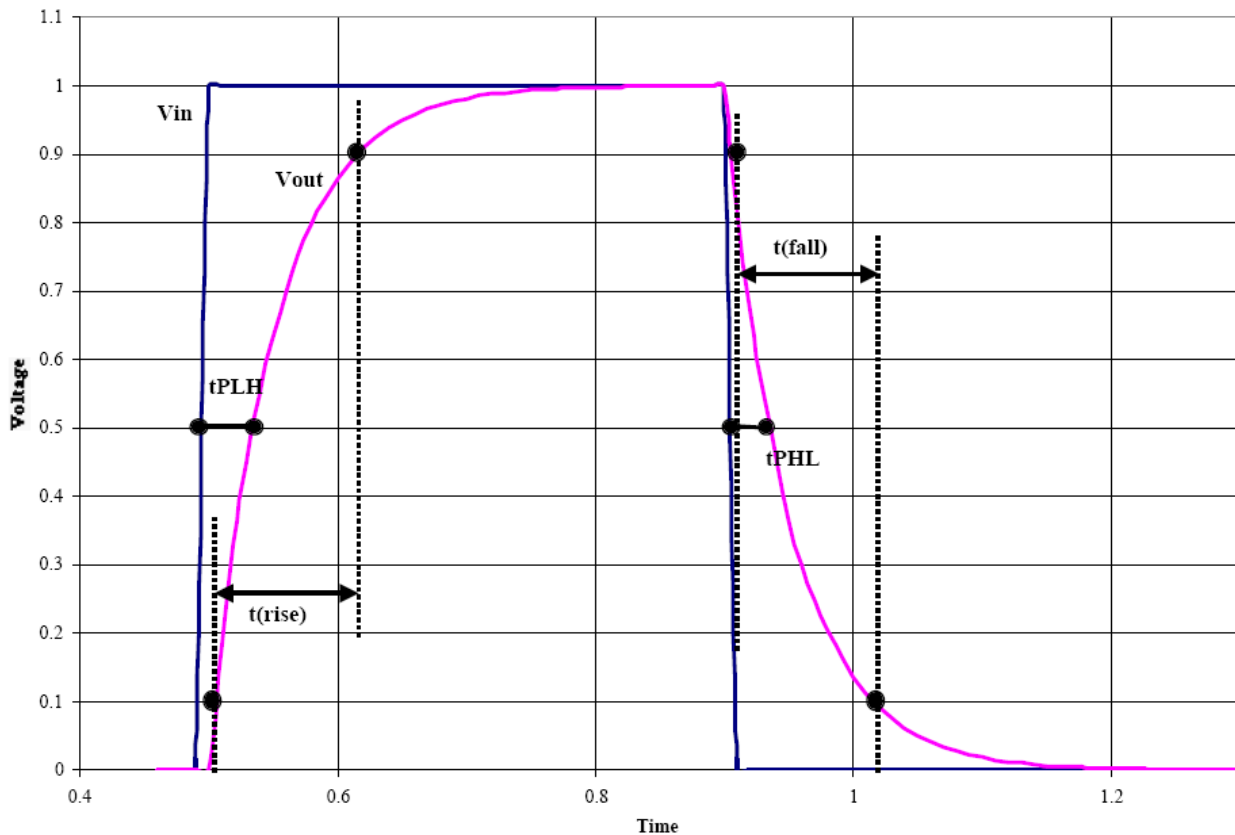


Figure 3. Timing parameters of signal waveforms.

The three timing parameters are defined as follows:

- Rise time: the time interval between the 10%-point and the 90%-point of the waveform when the signal makes the transition from low voltage (L) to high voltage (H). Notation: t_r .
- Fall time: the time interval between the 90%-point and the 10%-point of the waveform when the signal makes the transition from high voltage (H) to low voltage (L). Notation: t_f .
- Delay time (or propagation delay time): the time interval between the 50%-point of the input signal and the 50%-point of the output signal when both signals make a transition. There are two delay times depending on whether the output signal is going from L to H (delay notation t_{PLH}) or from H to L (delay notation t_{PHL}). The subscript P stands for “propagation.”

Note that the rise time and the fall time are defined using a single waveform (the output waveform) while the delay time is defined between two waveforms: the input waveform and the corresponding output waveform.

5.3 Parameter extraction via linear least-square-fit technique

The important parameters of $V_{out}(t)$ are the maximum amplitude and the time constant τ . The maximum amplitude is easily measured using the oscilloscope. To measure the time constant directly and accurately is more difficult since the waveform is an exponential function of time. A linear least-square-fit procedure can be used in the lab to extract the time constant from measured voltage

and time values as follows.

The equation for $V_{out}(t)$ during the time interval when $V_{out}(t)$ falls with time, which you can write based on what you learned in prerequisite courses, can be manipulated to provide a linear function in terms of the time t . The slope of this line is then used to extract the time constant τ .

Alternatively, the equation for $V_{out}(t)$ during the time interval when $V_{out}(t)$ rises with time can also be manipulated to provide a linear function in terms of the time t . The slope of this line is then used to extract the time constant τ .

In the lab, you will measure a set of data points (t, V_{out}) . These values, after the appropriate manipulation as above, can be used to plot a straight line, whose slope is a function of τ . You can use any procedure or a calculator to plot and extract the slope. The slope value will then be used to calculate the time constant τ .

Make sure you understand this procedure and be ready to use it in the lab. Note that the more points you measure, the more accurate the extracted value for τ .

5.4 Delay models of gates and interconnects using RC circuits

RC circuits are frequently used to model the timing characteristics of computer systems. When one logic gate drives another gate, the input circuit of the second gate can be modeled as an RC load. The propagation delay through the first gate can then be calculated assuming ideal square wave input and the RC load. The longer the delay time, the slower the circuit can be switched and the slower the computer is. Conversely, the shorter the delay time, the faster the computer is. This delay time is called “gate delay” since it relates to driving characteristics of a logic gate.

Another use of RC circuits is to model wiring characteristics of bus lines on integrated circuits (IC) or on printed-circuit boards (PCB). A wire can be modeled as many cascaded sections of simple RC circuits as shown in Figure 3 using 2 sections. When a square wave is applied to one end of the bus, it takes time for the signal to propagate to the other end. This delay time due to the wire can be calculated based on the values of R and C in each section and the number of sections used to model the wire. The longer the wire, the more sections are needed for accurate model. A wire is also referred to as “interconnect” and the delay due to a wire is also called “interconnect delay.” In high-frequency systems, the interconnect delay tends to dominate the gate delay and is a fundamental constraint on how fast a computer can operate.

6. Prelab

6.1 Equations for timing parameters of the step response

The input signal to the circuit in Figure 1 is a perfect square wave with amplitude A (from 0 V to A), and period T where $T \gg RC$. You may also assume that $R \gg R_s$ (the internal resistance of an arbitrary waveform generator). Using only symbolic parameters (e.g. R , C , A ; not numerical values), derive the equations for the following quantities:

- $V_{out}(t)$. What is the maximum value of $V_{out}(t)$? What is the minimum value of $V_{out}(t)$?
- The time values when the output reaches 10%, 50%, and 90% of its final value.

- c. Rise time t_r of $V_{out}(t)$.
- d. Fall time t_f of $V_{out}(t)$.
- e. Delay times t_{PHL} and t_{PLH} .

6.2 Parameter extraction via linear least-square-fit technique

Either technique below can be used to extract the time constant.

- a. From the equation for $V_{out}(t)$ during the time interval when $V_{out}(t)$ falls with time (see part 6.1.a above), write the equation for $\log\{V_{out}(t)\}$ as function of t . This equation should be linear in terms of t . Derive the equation for the slope of this line in terms of the time constant τ .
- b. Alternatively, from the equation for $V_{out}(t)$ during the time interval when $V_{out}(t)$ rises with time (see part 6.2.a above), manipulate this equation so that the final form looks like:

$$1 - \frac{V_{out}(t)}{A} = e^{-t/\tau}$$

where A is the amplitude of the step. Now you can write the equation for $\log\{1 - V_{out}(t)/A\}$ as function of t . This equation should be linear in terms of t . Derive the equation for the slope of this line in terms of the time constant τ .

6.3 Values of resistors

Use the digital multimeter (DMM) to measure the correct values of the resistors used in this lab. Record these measurements. Also record the internal resistance R_s of the arbitrary waveform generator when it is measured in the lab and used it whenever necessary.

7. Experimental procedures

7.1 Instruments needed for this experiment

The instruments needed for this experiment are: an arbitrary waveform generator, a multimeter, and an oscilloscope.

7.2 Step response of first-order RC circuits

1. Build the circuit in Figure 1 using $R = 10 \text{ K}\Omega$ and $C = 0.01 \text{ }\mu\text{F}$. Set the arbitrary waveform generator to provide a square wave input as follows:
 - a. Frequency = 300 HZ (to ensure that $T \gg RC$, $T=1/f$). This value of f guarantees that the output signal has sufficient time to reach a final value before the next input transition.
 - b. Set the amplitude to 3 Vpp and DC offset to 1.5V. Use the oscilloscope to display this waveform on Channel 1 to make sure the amplitude is correct. We use this amplitude since it is common in computer systems.
 - c. Set both channel 1 and channel 2 to DC coupling and probe gain X10
2. Use Channel 2 of the oscilloscope to display the output signal waveform. Do not select "on trigger" under trigger manual. Adjust the timebase to display 2 complete cycles of the signals. Use the cursor to measure and record the maximum and the minimum values of the output signal.

Turn in: Maximum and minimum

3. Measure the period T of the input signal, the time value of the 10%-point of V_{out} , the time value of the 90%-point of V_{out} , and the time value of the 50%-point of V_{out} .

Turn in: a table of time vs V_{out}

4. **Save screenshots** from the display with both waveforms and the measured values. Turn these screenshots in as part of your lab report.

Turn in: the screenshots

5. Set the timing cursor B to measure the rise time of V_{out} , the fall time of V_{out} , and the two delay times t_{PHL} and t_{PLH} between the input and output signals.

Turn in: a table of those times

6. **Save screenshots** from the display with both waveforms and the measured values. Turn these screenshots in as part of your lab report.

Turn in: the screenshots

7. Measure the voltage and time values at 10 points on the V_{out} waveform during one interval when V_{out} rises or falls with time (pick one interval only). Note that the time values should be referred to time $t = 0$ at the point where the input signal rises from 0 V to 3 V or falls from 3 V to 0 V. Record these 10 measurements.

Turn in: a table of time vs V_{out}

7.3 Step response of cascaded RC sections

1. Build the circuit in Figure 2, using 2 identical resistors $R = 10\text{ K}\Omega$ and 2 identical capacitors $C = 0.01\text{ }\mu\text{F}$. Use the function generator settings as in section 7.2 above and display it on Channel 1.
2. Display V_{out} on Channel 2 and adjust the timebase to display 2 complete cycles of the signals.
3. Set the timing cursor to measure the two delay times t_{PHL} and t_{PLH} between the input and output signals.

Turn in: a table of the measurement data

4. **Save screenshots** from the display with both waveforms and the measured values. Turn these screenshots in as part of your lab report.

Turn in: the screenshots

7.4 Manufacturing test time and test cost considerations

1. The more points you measure on a waveform, the more accurate the measured results but this also takes more time and increases the test cost. This is an important tradeoff in measurement accuracy and test cost. Given the circuit in Figure 1, ten data points per waveform were collected in section 7.2 item 7. Should you collect more or fewer than 10 points to extract a “good” estimate of the rise or fall time of the circuit (as defined above)? “Good” estimate means the estimated value is within 10% of the correct value (from computation or simulation).
2. What is the minimum number of data points do you need to collect to get a good estimate? Compare your minimum number of points with other teams’ answers. Do they collect more or

fewer points? Are their results “better” than yours? Include this discussion in your report.

8. Data analysis

8.1 Step response of first-order RC circuits

Use the component values in section 7.2 for the following data analysis. Remember to use the measured values of the external resistors in section 6.3 if they are significantly different than the marked values (more than 5%).

1. Using the numerical values for R and C in the Prelab equations in section 6.1 item b, compare the time values when the output voltage reaches the 10%-point, the 50%-point, and the 90%-point with the measured values in section 7.2 item 3. Explain the sources of errors leading to the differences. How much error is committed by neglecting the internal resistance of the arbitrary waveform generator?
2. Using the numerical values for R and C in the Prelab equations in section 6.1 items c, d, and e, compare the computed values of the 4 timing parameters (the rise time, the fall time, and the two delay times t_{PHL} and t_{PLH}) with the measured values in section 7.2 item 5. Explain the sources of errors leading to the differences. How much error is committed by neglecting the internal resistance of the arbitrary waveform generator?
3. From the 10 data points (V,t) measured in section 7.2 item 7, select 5 data points and use the best-fit line technique to extract the time constant τ of the output signal. Compare this value with the expected value RC and compute the difference in percent. Plot the best-fit line and the measured data points on the same plot.
4. Use all 10 data points in section 7.2 item 7, and repeat the best-fit line technique to extract the time constant. Is this new value different than the value computed in item 3 above by more than 5%? Plot the best-fit line and the measured data points on the same plot. Assuming that this new value for τ is the more accurate, use it to compute the correct value of the capacitor C. Compare the computed value of the capacitor with the marked value.

8.3 Step response of cascaded RC sections

1. From the measurements in section 7.3 item 2, are the delay times for the cascaded circuit in Figure 2 (of 2 identical RC sections) twice as large as the delay times for the simple RC circuit? Do the delay times scale with the number of sections? Explain.